Ableitungen nach der h-Methode auf der Basis des Differentialquotienten

Leiten Sie nach der h-Methode ab:

a) f (x) = x^2 b) f (x) = $3x^4$ c) f (x) = x^n

Lösung:

$$a)\quad f\left(x\right)=x^2$$

$$f'\left(x\right)=\lim_{h\to 0}\frac{f\left(x+h\right)-f\left(x\right)}{h}=\lim_{h\to 0}\frac{\left(x+h\right)^2-x^2}{h}$$

$$f'\left(x\right)=\lim_{h\to 0}\frac{x^2+2xh-h^2-x^2}{h}$$

$$f'\left(x\right)=\lim_{h\to 0}\frac{h\left(2x-h\right)}{h}=\lim_{h\to 0}\left(2x-h\right)\quad\xrightarrow{h\to 0}\quad 2x$$

Lösung:

$$b)\quad f\left(x\right)=3x^4$$

$$f'\left(x\right)=\lim_{h\to 0}\frac{f\left(x+h\right)-f\left(x\right)}{h}=\lim_{h\to 0}\frac{3\left(x+h\right)^4-3x^4}{h}$$

$$f'\left(x\right)=3\cdot\lim_{h\to 0}\frac{x^4+4x^3h+6x^2h^2+4xh^3+h^4-x^4}{h}$$

$$f'\left(x\right)=3\cdot\lim_{h\to 0}\frac{h\left(4x^3+6x^2h+4xh^2+h^3\right)}{h}$$

$$f'\left(x\right)=3\cdot\lim_{h\to 0}\left(4x^3+6x^2h+4xh^2+h^3\right)\quad\xrightarrow{h\to 0}\quad 12x^3$$

Lösung:

$$c) \quad f(x) = x^n$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^n - x^n}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\left[\sum_{k=0}^{n} \binom{n}{k} x^{n-k} h^k\right] - x^n}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{h\left(\sum_{k=1}^{n} \binom{n}{k} x^{n-k} h^{k-1}\right)}{h}$$

$$f'(x) = \lim_{h \to 0} \left[\binom{n}{1} x^{n-1} + \binom{n}{2} x^{n-2} h + \ldots + \binom{n}{n} h^{n-1}\right] \xrightarrow{h \to 0} n\, x^{n-1}$$